

Preheating with the Brakes On: The Effects of a Speed Limit

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We study preheating in models where the inflaton has a non-canonical kinetic term, containing powers of the usual kinetic energy. The inflaton field oscillating about its potential minimum acts as a driving force for particle production through parametric resonance. Non-canonical kinetic terms can impose a speed limit on the motion of the inflaton, modifying the oscillating inflaton profile. This has two important effects: it turns a smooth sinusoidal profile into a sharp saw-tooth, enhancing resonance, and it lengthens the period of oscillations, suppressing resonance. We show that the second effect dominates over the first, so that preheating with a non-canonical inflaton field is less efficient than with canonical kinetic terms, and that the expansion of the Universe suppresses resonance even further.

I. INTRODUCTION

After a sufficiently long period of inflation the Universe would be cold and devoid of observable matter. The energy responsible for driving inflation is trapped in the (nearly) homogeneous inflaton field ϕ . In order for observable matter to emerge from the post-inflationary Universe, the inflaton field must couple to additional degrees of freedom in a way that the inflationary energy is dumped into observable matter through a process known as reheating [1–3]. For the purpose of reheating, the inflaton can couple to other scalar fields, fermions, or gauge fields.

If the inflaton couples to bosonic fields, such as other scalar fields, novel condensation effects can take place. In particular, because there is no exclusion principle, the inflaton field can transfer a large amount of energy to the reheating field χ (the “reheaton”) in a process that is far from equilibrium. Such enhancements result from non-linear resonance effects due to the interaction between the inflaton and reheaton. For example, for the Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi) + \frac{1}{2}(\partial\chi)^2 + \frac{1}{2}g^2\phi^2\chi^2, \quad (1)$$

the equation of motion for large-scale modes of χ (neglecting the expansion of the Universe) becomes that of a harmonic oscillator with a time-dependent frequency,

$$\ddot{\chi}(t) + (k^2 + g^2\phi(t)^2)\chi(t) = 0 \quad (2)$$

where k is the comoving wavenumber of the χ field and $\phi = \phi(t)$ is the time-dependent background solution for the inflaton. At the end of single field inflation the inflaton oscillates about the minimum of its potential, so that the time-dependent frequency $\omega(t)^2 = k^2 + g^2\phi(t)^2$ oscillates with time. It is well-known that a harmonic oscillator with an oscillating time-dependent frequency can exhibit resonance effects, where the amplitude $\chi(t)$ of the oscillator grows exponentially with time. This post-inflationary exponential growth of certain long-wavelength modes of χ is dubbed “preheating”

[3–8], and has been studied in a variety of different contexts. See [9–11] for some reviews of this extensive literature. Most importantly for this paper, the majority of these studies (with the exception of [12]) have focused only on quadratic kinetic terms for the effective theory of reheating.

This approach towards inflationary and reheating model building neglects, however, the fact that these simple Lagrangians are really effective field theory (EFT) descriptions, only valid at sufficiently low energies. In particular, these Lagrangians should be understood as having been obtained by integrating out physics above some scale Λ at which new physics (such as new fields or new interactions) become important. The effects of physics above this energy can be parameterized in the EFT through non-renormalizable operators suppressed by powers of the scale of new physics,

$$\mathcal{L}_{eff} = \mathcal{L}_0 + \sum_{n>4} c_n \frac{\mathcal{O}_n}{\Lambda^{n-4}}. \quad (3)$$

As an example, consider the two-field Lagrangian,

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\rho)^2 + \frac{\rho}{\Lambda}(\partial\phi)^2 - \frac{1}{2}\Lambda^2\rho^2 - V(\phi), \quad (4)$$

with ϕ the inflaton field and ρ some heavy field with mass Λ . For energies below Λ , we can integrate out ρ at the classical (tree) level to obtain the effective Lagrangian [13, 14],

$$\mathcal{L}_{eff} = \frac{1}{2}(\partial\phi)^2 + \frac{(\partial\phi)^4}{\Lambda^4} - V(\phi). \quad (5)$$

The effective theory now contains a new contribution to the kinetic part of the action for the inflaton. More generally, one can consider as a low energy EFT a Lagrangian for the inflaton of the form¹

$$\mathcal{L}_{eff} = \mathcal{L}_{eff}(X, \phi), \quad (6)$$

¹ This is, of course, not the most general effective theory of the form (3). In particular, we have omitted terms involving higher

where $X \equiv -\frac{1}{2}(\partial\phi)^2$. Inflation with Lagrangians of this type can have novel features, such as a speed limit on the motion of the homogeneous inflaton and a sound speed of perturbations less than one $c_s^2 \leq 1$, that have important implications for inflationary perturbations and models [16–20].

It is the post-inflationary dynamics of (6) that is most interesting to us here. In particular, we will couple the non-canonical inflaton in (6) to a canonical reheaton field through a quartic interaction:

$$\mathcal{L}_{pre} = \mathcal{L}_{eff}(X, \phi) + \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}g^2\phi^2\chi^2 - \frac{1}{2}m_\chi^2\chi^2. \quad (7)$$

The equation of motion for the reheaton is still of the form (2). However, since the resonance arising from the time-dependent harmonic oscillator (2) is a non-linear effect, it is sensitive to the precise profile of the inflaton $\phi(t)$ as it oscillates about its potential minimum. The profile of the oscillation is in turn sensitive to the non-canonical kinetic terms in (7). Thus, the modified dynamics from non-canonical Lagrangians can play an important role in the physics of preheating.

In this paper, we will discuss the implications of an inflaton sector with non-canonical kinetic terms for preheating. We first discuss in Section II the dynamics of a non-canonical inflaton oscillating about its minimum. A fairly generic feature is the existence of a speed limit on the motion of the inflaton, restricting how fast it can move in field space. When the oscillating inflaton saturates this speed limit, its profile is no longer sinusoidal but instead takes a saw-tooth form. In Section III we show how the equation of motion for reheaton perturbations can be recast into a form of Hill’s equation, for which well-known techniques exist for finding resonance bands. Two competing effects lead to modifications of the standard theory of preheating: non-canonical kinetic terms lead to a “sharper” inflaton profile, enhancing resonance, while the period of oscillation is lengthened due to the speed limit, suppressing resonance. The net result is that the latter effect dominates, so that the resonance for non-canonical kinetic terms is *less efficient* than its canonical counterpart. After illustrating the effects of the expansion of the Universe, we summarize our results in Section IV. The Appendix contains further details about methods for computing the properties of parametric resonance.

derivatives. This can be done self-consistently as long as the higher derivatives are small for the physics we are interested in, which we will argue is the case. See also [13] for further analysis of the validity of the truncation (6) in the context of inflation. It is also worthwhile to note that certain Lagrangians of the form (6) are protected against corrections by powerful non-linear symmetries [15].

II. NON-CANONICAL KINETIC TERMS

Let us first focus on the implications that non-canonical kinetic terms of the form (6) have on the motion of the inflaton field oscillating about its minimum. Inflationary Lagrangians of this form with no potential energy have been proposed as an alternative to potential-dominated inflation, and are dubbed “k-inflation” [16]. More generally, however, we expect the inflationary sector to have both kinetic and potential energy; in this case, the effective Lagrangian $\mathcal{L}_{eff}(X, \phi)$ can be seen as an extension of standard slow-roll inflationary models with non-canonical kinetic terms. These non-canonical Lagrangians have a number of interesting properties that make them attractive to inflationary model building and phenomenology. On the model-building side, the non-canonical kinetic terms significantly modify the dynamics of the inflaton so that the speed of the inflaton can remain small even when rolling down a steep potential [13, 18]. In many cases, this results in a “speed limit” for the inflaton, namely a maximum for the speed of the homogeneous mode of the inflaton. This is a great advantage to model building, since then it is less necessary to fine-tune the inflationary potential to have very flat regions supporting slow roll inflation. Inflation with non-canonical kinetic terms can also lead to interesting signatures in the CMB, such as observable non-gaussianities [17, 19, 20].

We will choose to work with inflationary Lagrangians of the “separable” form (discussed in more detail in [13, 21]):

$$\mathcal{L}_{eff}(X, \phi) = \sum_{n \geq 0} c_n \frac{X^{n+1}}{\Lambda^{4n}} - V(\phi) = p(X) - V(\phi), \quad (8)$$

where we have written $p(X)$ as a power series expansion in X/Λ^4 , with Λ some UV energy scale. Since Λ is typically the mass scale of some heavy sector we have integrated out, we require it to be larger than the mass of the inflaton $\Lambda \gg m_\phi$ in order for such an effective field theory perspective to make sense. As a further restriction, we will take $c_0 = 1$, so that for small $X/\Lambda^4 \ll 1$, the Lagrangian reduces to the usual canonical Lagrangian. In order for the power series expansion to make sense, the series must have some non-zero radius of convergence R so that the series converges in the domain of convergence $X/\Lambda^4 \in [0, R)$, with $R \leq 1$ (it is not necessary that the series itself converge at the boundary). Finally, we will also require that the first and second derivatives of the power series are positive $\frac{\partial p}{\partial X}, \frac{\partial^2 p}{\partial X^2} > 0$ so as to guarantee that we satisfy the null energy condition and have subluminal propagation of perturbations [13, 22–24].

In a FLRW background

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 \quad (9)$$

the scale factor $a(t)$ is driven by the energy density of the

homogeneous inflaton $\phi(t)$,

$$\left(\frac{\dot{\phi}}{a}\right)^2 = \frac{1}{3M_p^2} \left[2X \frac{\partial p}{\partial X} - p(X) + V(\phi) \right], \quad (10)$$

where now $X = \frac{1}{2}\dot{\phi}^2$. The equation of motion for $\phi(t)$ in this background becomes,

$$\ddot{\phi} + 3H\dot{\phi}c_s^2 + \frac{\partial V}{\partial \phi} \frac{c_s^2}{\partial p/\partial X} = 0 \quad (11)$$

where the sound speed c_s^2 , defined by

$$c_s^2 = \left(1 + 2X \frac{\partial^2 p/\partial X^2}{\partial p/\partial X} \right)^{-1}, \quad (12)$$

is so called because it also appears as the effective speed of perturbations of ϕ about this background. We will ignore the expansion of the Universe for now, dropping the Hubble friction term in (11), and will return to the effects of expansion of the Universe in Section III C.

For simplicity, we will take the potential $V(\phi)$ of the inflaton to only consist of a mass term $V(\phi) = \frac{1}{2}m_\phi^2\phi^2$. This could be the entire inflaton potential, as in chaotic inflation, or just the form of the potential near its minimum. The difference is not particularly important, as we are primarily concerned with the phase when the homogeneous inflaton is oscillating about its minimum, where the quadratic form of the potential will be sufficient. For a scalar field with a canonical kinetic term in this potential, the equation of motion is that of a simple harmonic oscillator: the inflaton oscillates sinusoidally $\phi(t) = \Phi \sin(m_\phi t)$.

The behavior of a non-canonical kinetic term, however, can be qualitatively different. As with the canonical case, the potential provides a force that accelerates the inflaton. However, now the effective force $\frac{\partial V}{\partial \phi} \frac{c_s^2}{\partial p/\partial X}$ is also a function of the speed of the inflaton. Recall that the kinetic term $p(X)$ is a series which is only defined within a finite radius of convergence, $\frac{1}{2}\frac{\dot{\phi}^2}{\Lambda^4} \leq R$. As the inflaton speed approaches the radius of convergence $|\dot{\phi}| \rightarrow \sqrt{2R}\Lambda^2$ the series $p(X)$ or its derivatives $\partial p/\partial X, \partial^2 p/\partial X^2$ may converge or diverge, depending on the precise form of the series chosen. Notice that if the second derivative of the series $\partial^2 p/\partial X^2$ diverges faster than the first derivative $\partial p/\partial X$ at the boundary of the domain of convergence, so that $c_s^2 \rightarrow \frac{1}{2X} \frac{\partial p/\partial X}{\partial^2 p/\partial X^2} \rightarrow 0$, then the effective force vanishes. Said another way, as the force from the potential increases the speed of the inflaton, the non-canonical kinetic dynamics modify the *effective* force felt by the inflaton so that the effective force, and thus the acceleration of the inflaton, vanish as the inflaton approaches the radius of convergence. Thus, Lagrangians for which $\partial^2 p/\partial X^2$ diverges as $\dot{\phi}$ approaches the radius of convergence have a *speed limit*, such that $|\dot{\phi}| \leq \dot{\phi}_{max} = \sqrt{2R}\Lambda^2$. Certainly, as the inflaton approaches the speed limit, we are approaching the boundary of validity for the EFT

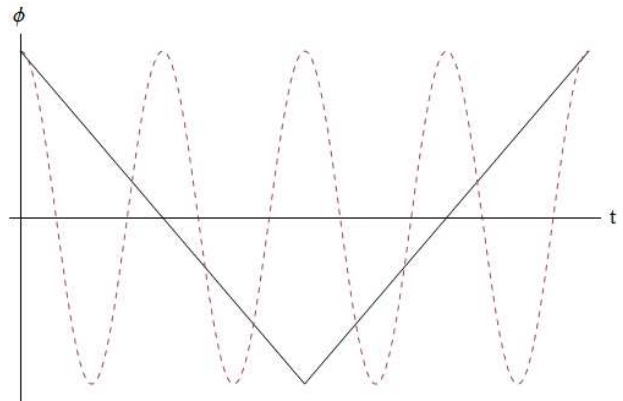


FIG. 1: The motion of the homogeneous inflaton $\phi(t)$ about the minimum of the (quadratic) potential differs depending on whether the kinetic terms are canonical or non-canonical. For canonical kinetic terms the motion is sinusoidal. For non-canonical kinetic terms the motion approaches a saw-tooth with a much longer period as the inflaton saturates its speed limit. This form of the inflaton profile is universal for Lagrangians that have a speed limit, except for a very small region near the turning point which is unimportant for preheating.

(8). Thus, it is advantageous to have a symmetry that protects the form of the Lagrangian against further corrections as this threshold is approached [15]. Further, requiring perturbativity of the inflaton perturbations places a bound on the minimal sound speed c_s^2 , thus restricting how close to the boundary of the EFT we can go [25–27]. Fortunately, there is a window where both the system is perturbative and the inflaton is close to its speed limit, so that the effects we are interested in below are still present.

For potentials that are sufficiently steep, the inflaton quickly attains its speed limit and stays there until it reaches the other side of the potential, where it decelerates and changes direction, as shown in Figure 1. This has two important consequences that will play a very important role in preheating:

- (a) The profile becomes sharper, with a smooth sinusoidal profile turning into a saw-tooth profile.
- (b) The speed limit slows the inflaton down, lengthening the period.

These two effects can be clearly seen in Figure 1. As mentioned earlier, the form of the potential about the minimum is in principle unrelated to the form of the potential during inflation. This implies that the behavior of the inflaton oscillating about its potential minimum can be dominated by the non-canonical kinetic terms, independent of whether inflation itself is dominated by these terms.

When the inflaton saturates the speed limit its profile

along one of the legs is approximately linear in time²:

$$\phi(t) \approx \sqrt{2R}\Lambda^2(t - t_j), \quad (13)$$

where t_j is the time when the inflaton crosses zero. For small Λ this linear approximation of the inflaton is valid up to the turning point $|\phi| \approx \Phi$, so the period of oscillation is

$$T_{NCR} = \frac{4\Phi}{\sqrt{2R}\Lambda^2}, \quad (14)$$

with $t \in [-\frac{T_{NCR}}{4}, \frac{T_{NCR}}{4}]$. In order for the non-canonical kinetic terms to be important, this must be much larger than the canonical period of oscillation $T_{CR} = 2\pi/m_\phi$ (otherwise the inflaton would not reach the speed limit during oscillation), so

$$T_{NCR} \gg T_{CR} \Rightarrow \Lambda \ll \left(\frac{2}{R}\right)^{1/4} \sqrt{\frac{\Phi m_\phi}{\pi}}. \quad (15)$$

This provides a precise condition for the small Λ limit where the system is very non-canonical. Together with the requirement that the effective field theory description make sense $\Lambda \gg m_\phi$, we have the following regime for Λ where both the EFT description makes sense and the speed limit is saturated:

$$1 \ll \frac{\Lambda}{m_\phi} \ll \left(\frac{2}{R}\right)^{1/4} \sqrt{\frac{\Phi}{\pi m_\phi}}. \quad (16)$$

Clearly, for too small of an amplitude Φ of the initial oscillation this cannot be satisfied. Since the expansion of the Universe and the transfer of energy from the inflationary to the reheating sector cause the amplitude of oscillation to decrease over time, eventually the inflaton no longer saturates the speed limit and the inflaton profile behavior returns to the canonical limit.

Perhaps the most well-known non-canonical Lagrangian that leads to a speed-limit for the inflaton is that of the DBI Lagrangian [18] (with constant warp factor, see [28]):

$$p(X) = -\Lambda^4 \left[\sqrt{1 - \frac{2X}{\Lambda^4}} - 1 \right]. \quad (17)$$

This Lagrangian arises by considering the motion of a space-filling D3-brane in a compact space, with ϕ taking the role of a transverse coordinate of the D3-brane. In this case the speed limit has a nice geometrical interpretation - it is just the effective speed of light for motion the extra dimensions (the effective speed of light is not necessarily unity if the metric on the internal space has

non-trivial warping). We can also represent (17) as a power series representation in powers of X/Λ^4 as in (8) with a radius of convergence $R = 1/2$. But the condition for obtaining the speed limit behavior can be satisfied by a much larger set of Lagrangians, not just the DBI Lagrangian. For example, Lagrangians of the form [13],

$$p(X) = -\Lambda^4 \left[\left(\left(1 - \frac{X}{\Lambda^4}\right)^R - 1 \right) \right], \quad (18)$$

with $R < 1$ (which includes (17) for $R = 1/2$), or

$$p(X) = -\Lambda^4 \left[\log \left(1 - \frac{X}{\Lambda^4} \right) - 1 \right] \quad (19)$$

are all of the form (8) (with radii of convergence $R, 1$ respectively) and all lead to speed-limiting behavior for ϕ . Importantly, though, the details of preheating driven by such fields will be insensitive to these different choices of Lagrangians - as long as there is a speed limit, the profile $\phi(t)$ will be that of the solid line in Figure 1.

III. PREHEATING WITH NON-CANONICAL INFLATION

A. Floquet Theory of Resonance

In the previous section we described the effects of non-canonical kinetic terms on the profile $\phi(t)$ of an inflaton oscillating about the minimum of its potential, ignoring the coupling of the inflaton to the reheaton field. Now, we will consider the impact on preheating of the coupling between the inflaton and reheaton sectors as in (7), with the inflationary Lagrangian given by (8),

$$\mathcal{L}_{pre} = p(X) - \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}m_\chi^2\chi^2 - \frac{1}{2}g^2\phi^2\chi^2 \quad (20)$$

where we will assume that $p(X)$ gives rise to a speed limit.

While the inflaton is dominated by its spatially homogeneous mode $\phi = \phi(t)$, which is oscillating with period T , the reheaton is assumed to have a vanishing background vacuum expectation value (VEV) $\langle\chi\rangle = 0$. Thus, we will consider fluctuations of the reheaton about the vacuum, which may be decomposed into a set of Fourier modes

$$\chi = \delta\hat{\chi}(\vec{x}, t) = \int \frac{d^3k}{(2\pi)^3} \left(e^{i\vec{k}\cdot\vec{x}} \delta\chi_k(t) + e^{-i\vec{k}\cdot\vec{x}} \delta\chi_k^*(t) \right). \quad (21)$$

Ignoring for now the expansion of the Universe, the equation of motion for the reheaton fluctuations is

$$m_\chi^2 \delta\chi_k(t) + (K + g^2\phi(t)^2) \delta\chi_k(t) = 0 \quad (22)$$

where $K \equiv k^2 + m_\chi^2$. This is easily recast into a driven harmonic oscillator known as Hill's equation [29], after

² This implies that the acceleration is approximately zero, so higher derivative corrections to the effective Lagrangian (8) are very small, as discussed in footnote 1.

a redefinition to a dimensionless time coordinate $\tau = 4\pi(t - t_0)/T$ (a prime denotes a derivative with respect to τ):

$$\delta\chi_k(\tau)'' + [A_k + qF(\tau)]\delta\chi_k(\tau) = 0, \quad (23)$$

where $F(\tau)$ is a 2π -periodic function symmetric about $\tau = 0$, satisfying $\int_{-\pi}^{\pi} F(\tau)d\tau = 0$. Floquet's theorem (see [29]) states that solutions to (23) have the form

$$\delta\chi_k(\tau) = e^{\tilde{\mu}_k\tau}g(\tau) + e^{-\tilde{\mu}_k\tau}g_2(\tau) \quad (24)$$

where $g(\tau), g_2(\tau)$ are oscillating solutions, and the Floquet growth exponent $\tilde{\mu}_k$ depends on A_k and q and is complex in general. For certain ranges of A_k and q , known as resonance bands, the real part of the Floquet growth exponent is non-zero³ leading to exponentially growing solutions:

$$\delta\chi_k(\tau) \sim e^{\tilde{\mu}_k\tau}. \quad (25)$$

In order to compare models of preheating with different oscillating profiles $\phi(t)$ it is more convenient to parameterize the growth exponent in terms of *physical time* t :

$$\delta\chi_k(t) = e^{\mu_k t} \quad (26)$$

where $\mu_k = \tilde{\mu}_k \frac{2\pi}{T}$, with T the period of oscillation of the inflaton. The relation between the two growth exponents is that $\tilde{\mu}_k$ represents the growth *per oscillation* of the inflaton, while μ_k represents that growth *per unit of physical time*, which takes into account effects on the overall growth due to changes in the period. The physical growth exponent μ_k is the appropriate quantity to use to evaluate the rate of growth compared to the rate of expansion of the Universe. Particle production is tracked by the number density n_k of particles, constructed as the energy of a mode divided by the effective mass $m_{eff,k}^2 = k^2 + m_\chi^2 + g^2\phi(t)^2$, and also scales with the Floquet exponent as

$$n_k = \frac{m_{eff,k}}{2} \left(\frac{|\delta\dot{\chi}_k(t)|^2}{m_{eff,k}^2} + |\delta\chi_k(t)|^2 \right) \sim e^{2\mu_k t}. \quad (27)$$

For any given oscillating profile $\phi(t)$ for the inflaton, with corresponding $F(\tau)$ in (23), it is possible to numerically determine the resonance bands; the procedure is outlined in Appendix A. We distinguish two opposite regimes:

- 1) **CR**: Canonical Reheating, where the reheaton is coupled to an inflaton with a canonical kinetic term (or equivalently where $\Lambda^2 \gg \dot{\phi}$ at all times so that the inflaton *effectively* behaves canonically), $p(X) \simeq X$.

- 2) **NCR**: Non-Canonical Reheating, where the reheaton is coupled to an inflaton with a non-canonical kinetic term $p(X)$, such that the inflaton speed approaches the speed limit $\dot{\phi} \simeq \sqrt{2R}\Lambda^2$ as it oscillates about its minimum.

First, let us write the equation of motion for the reheaton fluctuations $\delta\chi_k(t)$ in the form (23) for the CR case. As discussed in the previous section, an inflaton oscillating in a quadratic potential with a canonical kinetic term has a sinusoidal profile $\phi(t) = \Phi \sin(m_\phi t)$. The reheaton equation of motion can then be recast into the form of a Hill equation (23) with the identifications:

$$A_k = \frac{2K + g^2\Phi^2}{8m_\phi^2}, \quad q_{CR} = \frac{g^2\Phi^2}{8m_\phi^2}, \quad (28)$$

$$\tau = 2m_\phi t, \quad F(\tau) = \cos \tau \quad (29)$$

Hill's equation in this form is more commonly known as the Mathieu equation, and the resonance bands, plotted in (K, Φ) space in Figure 2, take the familiar form from previous studies of preheating [3, 6].

In the opposite limit (NCR), when the inflaton has a non-canonical kinetic term with a speed limit $|\dot{\phi}|_{max} = \sqrt{2R}\Lambda^2$, the profile $\phi(t)$ becomes a saw-tooth, as in Figure 1. Along one of the “legs” of this profile, the inflaton is linear in time as in (13), so that the reheaton equation of motion becomes:

$$\delta\ddot{\chi}_k(t) + (K + 2Rg^2\Lambda^4 t^2)\delta\chi_k(t) = 0. \quad (30)$$

This can also be rewritten in the form of a Hill's equation (23) with the identifications:

$$A_k = \frac{K\Phi^2}{2R\pi^2\Lambda^4} + \frac{1}{3} \frac{g^2\Phi^4}{2R\pi^2\Lambda^4}, \quad q_{NCR} = \frac{g^2\Phi^4}{2R\pi^2\Lambda^4},$$

$$\tau = \frac{\sqrt{2R}\Lambda^2\pi}{\Phi}t, \quad F(\tau) = \frac{\tau^2}{\pi^2} - \frac{1}{3}. \quad (31)$$

The resonance bands for this form of Hill's equation are shown in Figure 2. The two regimes can easily be connected numerically.

There are two main regimes of interest of (23) for the resonance bands: narrow resonance $q \ll 1$ and broad resonance $q \gg 1$. For $g^2\Phi^2/m_\phi^2 \ll 1$, the CR scenario is in the narrow resonance regime. When non-canonical kinetic terms are important, however, the parameter q_{NCR} in the corresponding Hill's equation (31) is enhanced relative to the canonical case

$$q_{NCR} = \frac{g^2}{2R\pi^2} \left(\frac{\Phi}{m_\phi} \right)^2 \frac{m_\phi^2\Phi^2}{\Lambda^4} = q_{CR} \frac{4}{R\pi^2} \frac{m_\phi^2\Phi^2}{\Lambda^4}. \quad (32)$$

The enhancement factor $m_\phi^2\Phi^2/\Lambda^4 \gg 1$ is large in order for the inflaton to saturate the speed limit, so unless q_{CR} is correspondingly small, when the CR scenario is in the narrow resonance regime the NCR scenario is in broad resonance. This has important physical implications, since then not only is the growth per period $\tilde{\mu}_k$

³ Without loss of generality, we will take the real part of the Floquet exponent to be positive.

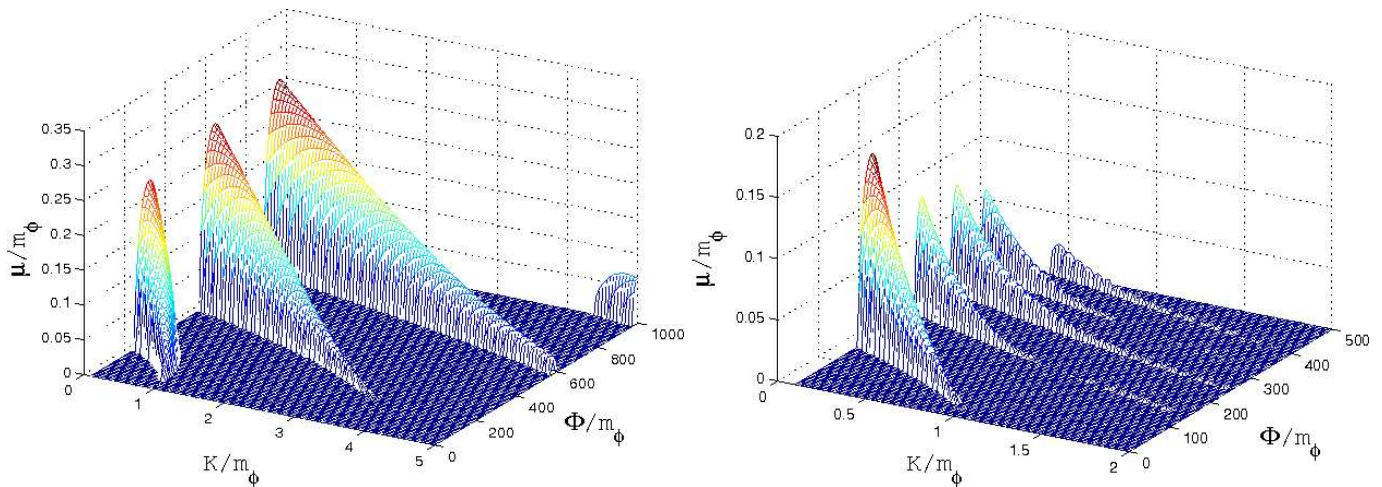


FIG. 2: Resonance bands for reheaton perturbations as a function of the scale K and initial amplitude of the oscillating inflaton Φ with $g = 0.005$ in the case when the reheaton is coupled to an inflaton with canonical (CR scenario, left) and non-canonical (NCR scenario, right) kinetic terms.

larger for non-canonical kinetic terms, but there is also growth over a much larger range of scales. We will see in the next section, however, that this enhancement is overwhelmed by suppression of particle production due to the lengthening of the period.

The resonance due to (22) depends on the physical wavenumber of the fluctuation k ; most particle production occurs when the effective mass $K + g^2\phi^2(t)$ vanishes. This implies that the resonance is most efficient at large scales $K \sim 0$, as can be seen in Figure 2. In practice, however, we cannot work on scales larger than the Hubble radius while neglecting metric perturbations, so $k > H$. In addition, a non-zero bare mass m_χ for the reheaton also keeps the effective mass from vanishing. But as long as we work on sufficiently large scales (and with a sufficiently large initial amplitude) so that $K \ll g^2\Phi^2$, the maximum of resonance at $K = 0$ will be a good approximation for the maximum resonance at large scales.

Figure 3 displays the growth exponent μ_k as a function of the initial amplitude Φ of the inflaton for large scales (namely $K = 0$). Several features are evident: for decreasing Λ , more resonance bands become accessible, because the system enters the broad resonance regime; the maximum size of the growth exponent in the first resonance band decreases as Λ decreases, reflecting the lengthening of the period; and the heights of the resonance bands for small Λ decrease with increasing Φ , also due to the lengthening of the period for large initial amplitudes (note that the period for canonical kinetic terms is independent of the amplitude).

This effect is not limited to the DBI case: Figure 4 compares the Floquet exponent for several Lagrangians of the forms (18) and (19). While the behaviors differ slightly as one moves away from the canonical case, they converge again in the regime where inflaton oscillations saturate the speed limit. The black dotted line in Figure

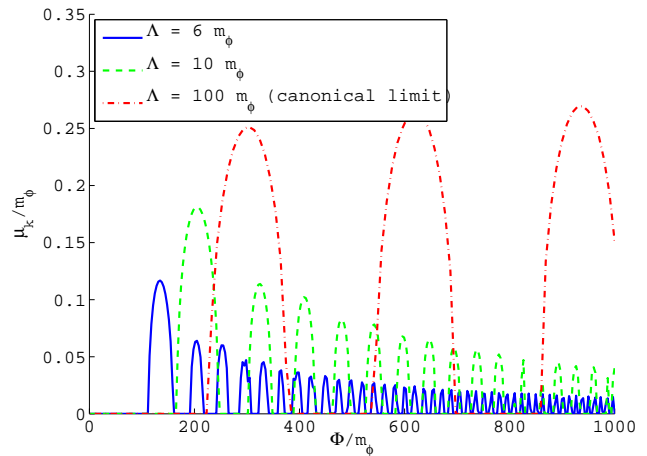


FIG. 3: Floquet growth exponent μ_0 per unit of physical time for $K = 0$ for three different values of Λ in DBI inflaton-driven preheating. Lowering the speed limit Λ^2 of the inflaton greatly reduces the strength of parametric resonance because the period of oscillation (14) increases as $1/\Lambda^2$. The additional suppression of the resonance due to the dependence of the period on the amplitude of inflaton oscillations Φ is also evident.

4 shows the Floquet exponent when the period lengthening is not taken into account: as the inflaton trajectory approaches the saw-tooth shape, energy is injected into the reheaton field over a longer amount of time, allowing for more particle production. This saturates when the slope $\dot{\phi}$ approaches the constant speed limit for most of the period. This enhancement is clearly subdominant, however, when compared with the suppression of particle production from the elongation of the period itself.

We close this section by comparing our results with

previous explorations in the literature. Ref. [12] found an expression for small $\dot{\phi}/\Lambda^2$ for the growth exponent μ_k in the case of a DBI inflaton by perturbing the canonical equation of motion. Up to order $\dot{\phi}/\Lambda^2$ and for $K = 0$, this corresponds to:

$$\mu_k \simeq \sqrt{\left(\frac{\theta_2}{2}\right)^2 - \left(\theta_0^{1/2} - 1\right)^2} \quad (33)$$

with

$$\theta_0 \equiv \frac{g^2 \Phi^2}{2m^2} \left(1 + \frac{9\Phi^2 m_\phi^2}{32\Lambda^4}\right), \quad \theta_2 \equiv \frac{g^2 \Phi^2}{4m^2} \left(1 + \frac{3\Phi^2 m_\phi^2}{8\Lambda^4}\right). \quad (34)$$

This result is illustrated in the right panel of Fig. 4. For $m_\phi/\Lambda < 1$ it provides a good approximation of the gain in particle production one would expect from DBI inflation were it not for the lengthening of the period, which the authors of [12] correctly identified as “DBI friction” but did not quantify. We are interested in the opposite limit, however, where the non-canonical kinetic terms are more than just a small perturbation.

B. Non-Canonical Limit

In the non-canonical limit, the equation of motion for the reheaton perturbations (30) can be written in a simple and suggestive form by making a redefinition to a different dimensionless time coordinate than we considered earlier:

$$\tau = \left(g\sqrt{2R}\Lambda^2\right)^{1/2} t = (2Rg^2)^{1/4} \Lambda t. \quad (35)$$

The reheaton equation of motion then becomes

$$\frac{d^2 \chi_k(\tau)}{d\tau^2} + [\kappa^2 + \tau^2] \chi_k(\tau) = 0, \quad (36)$$

where $\kappa^2 \equiv \frac{k^2 + m^2}{\sqrt{2R}g\Lambda^2}$. The time range of the new time variable is $-\frac{\Delta\tau}{2} \leq \tau \leq \frac{\Delta\tau}{2}$ where $\Delta\tau = T_\Lambda(2g^2R)^{1/4}\Lambda = (8g^2/R)^{1/4}(\Phi/\Lambda)$.

The benefit of making this redefinition is that the problem of broad resonance can be mapped to the problem of scattering of a particle with energy κ^2 off of an inverted parabolic potential with an effective Schrödinger equation [3, 6]:

$$\frac{d^2 \chi_k(\tau)}{d\tau^2} + (\kappa^2 - V_{eff}(\tau))\chi_k(\tau) = 0. \quad (37)$$

This scattering problem can be solved using standard techniques (see Appendix B), leading to the result:

$$\tilde{\mu}_k = \frac{1}{2\pi} \ln \left[\frac{1 + |R_k|^2}{|D_k|^2} + 2 \frac{|R_k|}{|D_k|^2} \cos \theta_{tot} \right] \quad (38)$$

in terms of the reflection and transmission amplitudes R_k, D_k and the phase of the reheaton mode θ_{tot} . For an

inverted parabolic potential, the reflection and transmission coefficients are known, so the average growth index simplifies to be [3]:

$$\tilde{\mu}_k = \frac{1}{2\pi} \ln \left[1 + 2e^{-\pi\kappa^2} - 2 \sin \theta_{tot} e^{-\frac{\pi}{2}\kappa^2} \sqrt{1 + e^{-\pi\kappa^2}} \right] \quad (39)$$

For large scales $\kappa^2 \approx 0$ this has a maximum/average value of $\tilde{\mu}_k \approx (0.28, 0.175)$ [3].

The average growth index $\tilde{\mu}_k \approx 0.175$ only gives the typical growth over one period of oscillation for broad resonance, and is the same regardless of whether the inflaton dynamics are canonical or non-canonical. In particular, it does not depend on the type of non-canonical Lagrangian used - *any* non-canonical Lagrangian that leads to a speed limit for the oscillating inflaton will have the same average growth factor over one period $\tilde{\mu}_k$. The reason the average growth index does not depend on whether the system is canonical or non-canonical is because we are working in the broad resonance limit, where only the slope of the inflaton profile (in dimensionless coordinates) at the point where the inflaton crosses zero matters.

However, as discussed before, the true comparison of the particle production between the canonical and non-canonical models should be the growth μ_k over some fixed *physical* time period Δt

$$n_k \propto e^{2\tilde{\mu}_k \frac{2\pi}{T} \Delta t} \sim e^{2\mu_k \Delta t}, \quad (40)$$

where T is the period, so that the true growth index is the average growth index over one period divided by the length of the period, $\mu_k = \tilde{\mu}_k 2\pi/T$. This leads to the main result of this section: since the period for an oscillating inflaton with non-canonical dynamics is much longer than the period for an oscillating inflation with canonical dynamics $T_{NCR} \gg T_{CR}$, we have *much more growth in a canonical system* over some fixed physical time Δt , $(\mu_k)_{CR} \gg (\mu_k)_{NCR}$. In particular, the ratio between the two should fall off as the ratio of their periods,

$$\frac{(\mu_k)_{NCR}}{(\mu_k)_{CR}} = \sqrt{\frac{R}{2}} \frac{\pi \Lambda^2}{\Phi m_\phi} \ll 1, \quad (41)$$

for decreasing Λ at $\kappa = 0$. Indeed, this is precisely what is found using numerical techniques, as shown in Figure 4.

Finally, we comment on the possibility for non-canonical kinetic terms to enhance resonance by turning a putative CR system in narrow resonance into a NCR system with broad resonance. A CR system in narrow resonance with $q_{CR} \ll 1$ has $(\mu_k)_{CR} \sim q_{CR} m_\phi$ [3]. As discussed in Section III A, the same system in NCR has $q_{NCR} \sim q_{CR} f^2$, with $f = \Phi m_\phi / (\sqrt{2R}\Lambda^2) \gg 1$. But as just discussed the growth exponent in broad resonance for NCR is a fixed number divided by this enhancement factor, $(\mu_k)_{NCR} \sim 0.1 m_\phi / f$. Thus, in order for the NCR system to be in broad resonance $q_{NCR} \gtrsim 1$, we have that $(\mu_k)_{NCR} \lesssim 0.1 m_\phi q_{CR}^{1/2}$, representing an enhancement

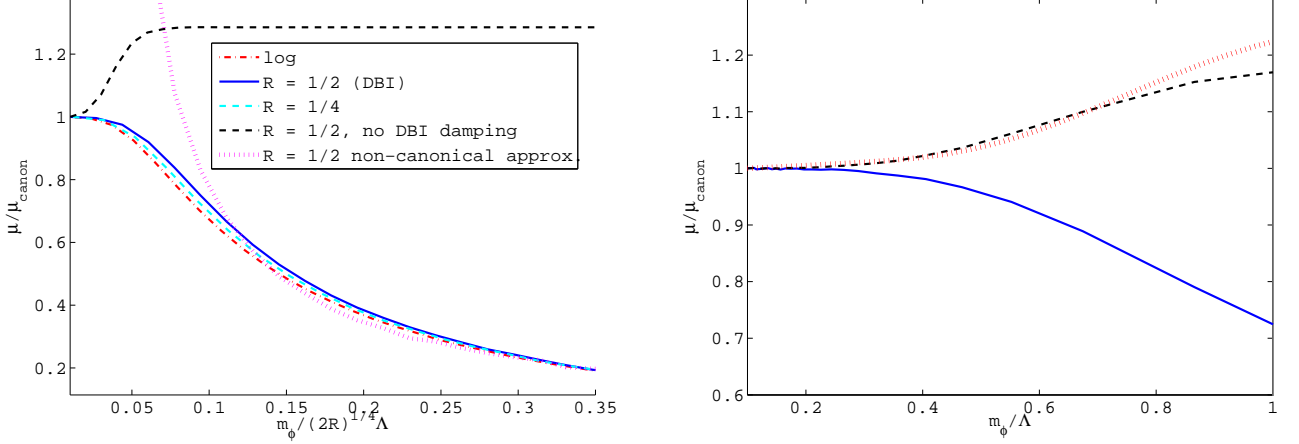


FIG. 4: Floquet exponent μ of the first resonance band for $K = 0$ for $g = 0.005$ (left) and $g = 0.5$ (right). $\Lambda \gg m_\phi$ corresponds to the canonical case CR. Parameter R in the left-hand panel refers to (18), and log refers to (19). In both panels the black dashed line is the growth of particle production per period, whereas the blue solid line is the true growth of particle production per physical unit of time. Additionally, in the left panel the magenta dotted line is the analytical result (41), showing good agreement with the numerical results in the regime where the speed limit is important. In the right panel we have also included the approximate result from [12] (dotted red line, (33)), which did not include the suppression from the speed limit. As discussed in Section II, the effective theory breaks down for $m_\phi > \Lambda$ and results should therefore not be trusted beyond this.

from CR narrow resonance by a factor of $0.1/q_{CR}^{1/2}$. While this enhancement can be large if the original CR system is very far in the narrow resonance regime, the growth factor is still quite suppressed.

C. Expansion of the Universe

Up to now we have ignored the expansion of the Universe. Including expansion has several effects on preheating which have been well-studied for CR [3, 6, 8]:

- i) The amplitude of inflaton oscillations decreases due to Hubble damping inversely proportional to time $\Phi(t) \sim t^{-1}$.
- ii) The parameters of Hill's equation (A_k, q) become time dependent. In particular, $q_{CR} \sim t^{-2}$ due to the time dependence of the inflaton amplitude.
- iii) The physical wavenumber $k_{phys} = k/a(t)$ redshifts with the expansion, so each comoving mode only spends a finite amount of time in a resonance band.
- iv) The resonance is *stochastic* [3].
- v) Preheating is ineffective when the rate of particle production drops below the expansion rate, e.g. when $\mu_k < H$. For narrow resonance this condition cannot be satisfied, thus particle production can only occur in the broad resonance regime.

We will now make some simple estimates of how these effects will change for NCR.

First, the Hubble friction term may be removed from the inflaton equation of motion (11) by rescaling the inflaton field by powers of the scale factor $\phi(t) = \Phi_0 \tilde{\phi}(t)/a(t)^{3/2c_s^2}$, so that $\tilde{\phi}(t)$ is a fixed unit-amplitude oscillating function, and $\Phi(t) = \Phi_0/a(t)^{3/2c_s^2}$ is the time-dependent amplitude. When the period of oscillation of the inflaton is much smaller than the timescale for the expansion of the Universe, the energy density can be averaged over several oscillations $\rho_\phi \sim m_\phi^2 \phi^2 \sim a(t)^{-3/c_s^2}$. The scale factor thus grows as $a(t) \sim t^{2/3c_s^2}$, so that the amplitude falls is inversely proportional to time $\Phi(t) \sim \Phi_0/t$ as with a canonical kinetic term. The decaying inflaton profile in an expanding background is shown in Figure 5. It is worth noting that in the far non-canonical limit the period of the inflaton can be larger than the Hubble time due to the period lengthening effect. When this occurs, Hubble friction damps a significant amount of the amplitude in the first oscillation, quickly bringing the system back to the previous case.

For a fixed cutoff scale Λ , the condition for the oscillating inflaton to saturate the speed limit (16) is violated once the amplitude of oscillation drops below some critical amplitude. The number of oscillations of the inflaton scales with time $N \sim t/T$, so that the time-dependent amplitude drops below this critical amplitude after $N \sim m_\phi \Phi_0/\Lambda^2$ oscillations, where Φ_0 is the initial amplitude. Thus the inflaton no longer saturates the speed limit after $N \sim m_\phi \Phi_0/\Lambda^2$ oscillations, and behaves canonically after this time.

Another important effect is the dependence of the Hill's equation parameters (31) on time. Since q_{NCR} depends on the fourth power of the amplitude, it quickly

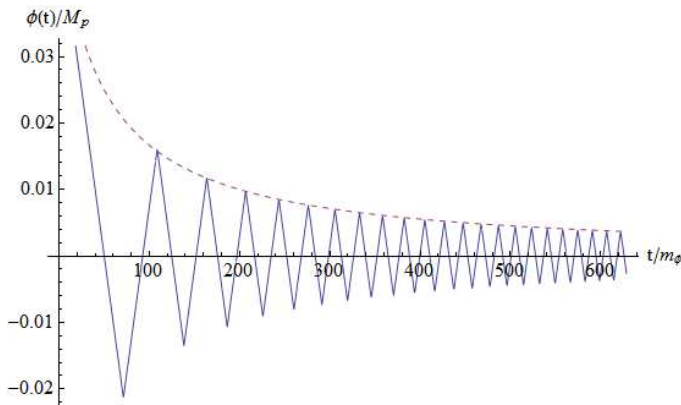


FIG. 5: Expansion of the Universe causes the amplitude of the inflaton to decay inversely proportional to time, as shown here for $\Lambda = 10 m_\phi$ and $m_\phi = 10^{-5} M_P$ with an initial amplitude of $\Phi_0 = 0.05 M_P$. The dashed line indicates the t^{-1} behavior of the decaying amplitude.

becomes small at a much faster rate $q_{NCR} \sim t^{-4}$ than it would in CR, $q_{CR} \sim t^{-2}$. In particular, after only $N \sim g^{1/2} \Phi_0 / \Lambda$ oscillations, the narrow resonance regime where $q_{NCR} \lesssim 1$ is attained. As noted above, narrow resonance with CR is inefficient for an expanding background. Since we argued in the previous section that particle production for NCR is suppressed compared to CR, narrow resonance for NCR is also inefficient in an expanding background. This means that after $N \sim g^{1/2} \Phi_0 / \Lambda$ oscillations the resonance shuts off due to expansion of the Universe. The other effects of an expanding Universe listed above for CR (iii,iv) do not appear to be qualitatively different for NCR, so we will not comment further on them.

In summary, including expansion of the Universe for NCR leads to a decaying amplitude of inflaton oscillations inversely proportional to time $\Phi(t) \sim t^{-1}$. After many oscillations the amplitude decreases so much that either: (a) the inflaton ceases to saturate the speed limit during oscillation, effectively becoming canonical; or (b) the Hill's equation parameter $q_{NCR}(t)$ becomes so small that particle production cannot compete with the expansion of the Universe, shutting off the resonance. Which outcome dominates depends on the relative magnitudes of m_ϕ / Λ and $g^{1/2}$; when the former dominates, outcome (a) occurs first, and *vice-versa*.

IV. CONCLUSION

Preheating in the post-inflationary Universe is the explosive production of particles far from thermal equilibrium that occurs as the inflaton field oscillates about its potential minimum. We have examined preheating for an inflaton sector that has non-canonical kinetic terms arising as an effective theory $\mathcal{L}_{eff}(X, \phi)$, for $X = \frac{1}{2}(\partial\phi)^2$, which may arise from the existence of new physics at

some energy scale $\Lambda > m_\phi$. Effective theories of this type can give rise to a speed limit for the motion of the inflaton ϕ , as in DBI inflation [18]. In addition to having important implications for inflationary model building, the speed limit plays an important role in modifying the nature of preheating in the post-inflationary Universe. In particular, as the non-canonical terms become important, non-canonical preheating departs significantly from the canonical case via three main effects:

- i) The sinusoidal inflaton profile becomes a saw-tooth, elongating the fraction of the inflaton period in which significant particle production may occur, and moves the system from narrow to broad resonance.
- ii) Effect i) is offset by an elongation of the inflaton oscillation period by a factor $f = \sqrt{\frac{2}{R} \frac{\Phi m_\phi}{\pi \Lambda^2}} \gg 1$. This suppresses the amount of χ particle production *per unit time* by $1/f$.
- iii) Effect ii) affects the competition between χ production and Hubble expansion, making preheating even less efficient in an expanding Universe.

In general, then, preheating when the inflation has non-canonical kinetic terms is *less efficient* than with a purely canonical kinetic term. This implies that if preheating is to be important at all in the early Universe, then the UV scale of new physics that couples kinetically to the inflaton must be sufficiently high that the effective non-canonical kinetic terms are negligible.

We have only focused on non-canonical kinetic terms for the inflaton sector in this paper. Certainly, however, due to the non-linear nature of the parametric resonance, it would be interesting to study how non-canonical kinetic terms for the reheating sector would affect the physics of preheating. We leave this to future work.

Acknowledgments

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Appendix A: Computing the Floquet exponent

There exists a straightforward method of computing the Floquet exponent μ_k as a function of the parameters A_k and q , which has been extensively covered in the literature (e.g. [29–31]).

We start with Hill's equation of the form (23). The periodic function $F(\tau)$ may be decomposed:

$$F(\tau) = \sum_{n=-M}^M d_n e^{in\tau}, \quad (\text{A1})$$

with A_k defined such that:

$$\int_{-\pi}^{\pi} F(\tau) d\tau = 0. \quad (\text{A2})$$

Floquet's theorem (see [29]) states that solutions to (23) are of the form,

$$\chi_k(\tau) = e^{\tilde{\mu}_k \tau} g(\tau) + e^{-\tilde{\mu}_k \tau} g_2(\tau) \quad (\text{A3})$$

where $g(\tau)$ and $g_2(\tau)$ are periodic functions with period T , and $\tilde{\mu}_k$, called the *Floquet exponent* or *characteristic exponent*, is complex. Clearly when $\tilde{\mu}_k$ has a non-zero real part we have exponential growth of χ_k — this is the parametric resonance effect. Without loss of generality, we will take the real part of $\tilde{\mu}_k$ to be positive, and so we will drop $g_2(\tau)$ since its coefficient is exponentially decreasing.

In order to find solutions of the form (A3), we first Fourier expand $g(\tau)$

$$\chi_k(\tau) = \sum_{n=-\infty}^{\infty} c_n e^{(\tilde{\mu}_k + in)\tau} \quad (\text{A4})$$

and plug this Fourier series back into the equation of motion (23) to derive a recursion relation for the coefficients c_n in terms of $\tilde{\mu}_k, A_k, q, d_n$:

$$c_n + \frac{q \sum_{m=-M}^M d_m c_{n-m}}{((\tilde{\mu}_k + in)^2 + A_k)} = 0 \quad \forall n \in (-\infty, \infty). \quad (\text{A5})$$

These recursion relations define a matrix problem

$$B(\tilde{\mu}_k, A_k, q, d_m) \begin{pmatrix} c_{-n} \\ \vdots \\ c_{-1} \\ c_0 \\ c_1 \\ \vdots \\ c_n \end{pmatrix} = 0 \quad (\text{A6})$$

with $n \rightarrow \infty$, where the elements B_{rs} of the (infinite) matrix B are given by:

$$B_{rs} = \begin{cases} 1 & \text{if } r = s \\ \frac{q d_{r-s}}{(\tilde{\mu}_k + ir)^2 + A_k} & r \neq s \end{cases}. \quad (\text{A7})$$

The matrix problem (A6) implies that the (infinite) matrix B is singular, so it must have vanishing determinant

$$\Delta(\tilde{\mu}_k, A_k, q, d_m) = |B_{rs}| = 0. \quad (\text{A8})$$

The vanishing of the determinant (A8) defines an implicit function $\tilde{\mu}_k = \tilde{\mu}_k(A_k, q, d_m)$. In practice, we perform a

Fourier transform of the inflaton profile written in terms of $F(\tau)$ numerically, and evaluate the $M \times M$ determinant (A8); a matrix size $M \sim 100$ is sufficient for convergence of the first tens of resonance bands.

Let us briefly comment on the differences in solving for the growth exponent $\tilde{\mu}_k$ using this method for canonical and non-canonical kinetic terms. Clearly, $F(\tau)$ for non-canonical kinetic terms (31) expanded in Fourier modes involves many terms, as opposed to the single Fourier mode for canonical kinetic terms. Thus, one difference between resonance with canonical and non-canonical kinetic terms is the inclusion of more terms in the Fourier expansion of the profile. Naively, then, there are more modes in the driving force with which the χ_k fields may resonate. This is the feature arising from the “sharpening” of the profile. However, the other important effect of the non-canonical kinetic terms is the extreme lengthening of the period of oscillation, thus suppressing the resonance in physical time.

Appendix B: Scattering from a Parabolic Potential

In many cases of interest, the equation for perturbations of the reheaton χ_k :

$$\ddot{\chi}_k(t) + [k^2 + m_\chi^2 + g^2 \phi^2] \chi_k(t) = 0, \quad (\text{B1})$$

can be converted into a Schrödinger scattering problem. In this appendix, we will outline the equivalence between these two perspectives; see [6] for more on the correspondence. We will imagine that $\phi(t)$ has a large slope in some region for some finite amount of time, as shown in Figure 6. Linearizing about this point, we have

$$\phi \approx \phi_0 + \lambda(t - t_i). \quad (\text{B2})$$

This linear approximation is valid roughly for some time interval $2\Delta t$, i.e. for $t_i - \Delta t \leq t \leq t_i + \Delta t$. Outside this interval, the field χ must behave adiabatically: its potential is slowly-varying and can therefore be approximated by a WKB approach. The equation of motion around the point of adiabaticity violation becomes:

$$\ddot{\chi}_k(t) + [k^2 + m_\chi^2 + g^2(\phi_0 + \lambda(t - t_i))^2] \chi_k(t) = 0. \quad (\text{B3})$$

We can map this problem into a Schrödinger-like equation by redefining the time variable $\tau' \equiv (\frac{g}{\lambda})^{1/2}(\phi_0 + \lambda(t - t_i))$, so that the equation of motion is written:

$$\frac{d^2 \chi_k(\tau')}{d\tau'^2} + [\kappa^2 + \tau'^2] \chi_k(\tau') = 0. \quad (\text{B4})$$

We have defined $\kappa^2 \equiv \frac{k^2 + m_\chi^2}{\lambda g}$. The time range of the new variable is $\tau'_i - \Delta\tau' \leq \tau' \leq \tau'_i + \Delta\tau'$, where $\tau'_i = \phi_0(g/\lambda)^{1/2}$ and $\Delta\tau' = (\lambda g)^{1/2} \Delta t$. The transformed equation (B4) resembles a Schrödinger equation with the effective potential:

$$V_{eff} = \begin{cases} -(\tau'_i - \Delta\tau')^2 = -V_0 = \text{const} & \tau' < \tau'_i - \Delta\tau' \text{ (Region I)} \\ -\tau'^2 & \tau'_i - \Delta\tau' \leq \tau' \leq \tau'_i + \Delta\tau' \text{ (Region II)} \\ -(\tau'_i + \Delta\tau')^2 = -V_1 = \text{const} & \tau' > \tau'_i + \Delta\tau' \text{ (Region III)} \end{cases} \quad (\text{B5})$$

The setup of this effective scattering problem is shown in Figure 6.

The solution in each region is therefore known:

$$\chi_k = \begin{cases} A_1 e^{-ik_1\tau'} + B_1 e^{ik_1\tau'} & \text{(Region I)} \\ A_2 D_\nu((1+i)\tau') + B_2 D_\nu((-1+i)\tau') & \text{(Region II)} \\ A_3 e^{-ik_2\tau'} + B_3 e^{ik_2\tau'} & \text{(Region III)} \end{cases} \quad (\text{B6})$$

where $k_1 \equiv \sqrt{\kappa^2 + V_0^2}$, $k_2 \equiv \sqrt{\kappa^2 + V_1^2}$, $\nu \equiv \frac{1}{2}i(i - \kappa^2)$, and the functions $D_\nu(x)$ are *parabolic cylinder functions*.

In the language of scattering matrices, the ingoing/outgoing waves are parameterized as

$$\begin{pmatrix} \alpha_k^{j+1} e^{-i\theta_j^k} \\ \beta_k^{j+1} e^{i\theta_j^k} \end{pmatrix} = \begin{pmatrix} \frac{1}{D_k} & \frac{R_k^*}{D_k^*} \\ \frac{R_k}{D_k} & \frac{1}{D_k^*} \end{pmatrix} \begin{pmatrix} \alpha_k^j e^{-i\theta_j^k} \\ \beta_k^j e^{i\theta_j^k} \end{pmatrix} \quad (\text{B7})$$

where D_k, R_k are the transmission and reflection coefficients, the α_k, β_k are normalized such that $|\alpha_k|^2 - |\beta_k|^2 = 1$ with the “particle number” defined as $n_k = |\beta_k|^2$, and $\theta_k^j \equiv \int_0^T \sqrt{k^2 + m_\chi^2 + g^2\phi^2} dt$ is the accumulated WKB phase up to the time of scattering. Apart from an overall normalization, it should be clear that the α^j ’s correspond to the A_j ’s and the β^j ’s to the B_j ’s above.

From (B7) we can write

$$\beta_k^{j+1} = \alpha_k^j e^{-2i\theta_j^k} \frac{R_k}{D_k} + \beta_k^j \frac{1}{D_k} \quad (\text{B8})$$

which we can use to find the final number of particles n_k^{j+1} in terms of the original number of particles n_k^j (together with the normalization condition on the α):

$$n_k^{j+1} = \left| \frac{R_k}{D_k} \right|^2 + \frac{1 + |R_k|^2}{|D_k|^2} n_k^j + 2 \frac{|R_k|}{|D_k|^2} \cos \theta_{tot} \sqrt{(1 + n_k^j) n_k^j} \quad (\text{B9})$$

where $\theta_{tot} \equiv \arg \alpha_k^j - \arg \beta_k^j - \arg R_k - 2\theta_k^j$ is the total accumulated phase. In the large n_k^j limit, we can drop the first term and simplify the last term, so that we have

$$n_k^{j+1} = \left[\frac{1 + |R_k|^2}{|D_k|^2} + 2 \frac{|R_k|}{|D_k|^2} \cos \theta_{tot} \right] n_k^j. \quad (\text{B10})$$

The average growth exponent over one period is defined as

$$n_k^{j+1} = e^{2\tilde{\mu}_k \frac{\pi}{T} \Delta t} n_k^j = e^{2\pi \tilde{\mu}_k} n_k^j \quad (\text{B11})$$

for $\Delta t = T$ over one period. Thus, the growth exponent is,

$$\tilde{\mu}_k = \frac{1}{2\pi} \log \left[\frac{1 + |R_k|^2}{|D_k|^2} + 2 \frac{|R_k|}{|D_k|^2} \cos \theta_{tot} \right]. \quad (\text{B12})$$

Averaging over many possible initial phases, we can consider θ_{tot} as a random variable; thus, we will take $\cos \theta_{tot} \sim 0$ on average.

In order to compute the scattering amplitudes R_k, D_k , one needs to solve the continuity equations across the boundaries of Regions I, II and III for the wavefunctions (B6). In general, it is not possible to solve these equations analytically. However, for special cases the equations can in fact be solved.

The first limit we will take is $|\tau'_i| \gg 1$; in the original setup, this corresponds to taking $\phi_0 \sqrt{g}/\sqrt{\lambda}$ to be large, e.g. the region of linearization takes place very far from $\phi = 0$. In this limit the effective potential becomes essentially a step function:

$$V_{eff} = \begin{cases} -V_0 & \tau' < \tau'_i \text{ (Region I)} \\ -V_1 & \tau' > \tau'_i \text{ (Region II)} \end{cases} \quad (\text{B13})$$

where $|V_0 - V_1| = \sqrt{g/\lambda} |\Delta\phi|$, with $\Delta\phi$ the change in the inflaton over the time period Δt , and $|V_0| \sim |V_1| \gg 1$. In order for the step function approximation to be a valid approximation to the inverted quadratic effective potential, we need that the change over the step function be small, e.g. $\frac{|V_0 - V_1|}{V_1} \ll 1$. The transmission and reflection coefficients can easily be computed

$$D_k^2 = \frac{4(\kappa^2 + V_0)(\kappa^2 + V_1)}{(2\kappa^2 + V_0 + V_1)^2}; \quad R_k^2 = \frac{(V_0 - V_1)^2}{(2\kappa^2 + V_0 + V_1)^2}; \quad (\text{B14})$$

so that the growth exponent becomes

$$\tilde{\mu}_k = \frac{1}{2\pi} \ln \left[\frac{1 + (V_0 - V_1)^2}{4\sqrt{(\kappa^2 + V_0)(\kappa^2 + V_1)}} \right]. \quad (\text{B15})$$

With the requirements above that the size of the step be small, we have $D_k \approx 1$ and $R_k \approx 0$, so that $\tilde{\mu}_k \sim \ln(1) \sim 0$. This is expected - if the inflation has a very large offset, the effective mass of the reheaton $m_{eff}^2 = k^2 + m_\chi^2 + g^2\phi_0^2$ is large, so there will be very little particle production.

Alternatively, in the limit $\tau'_i = 0$, the scattering potential becomes symmetric about $\tau = 0$. For $\Delta\tau \gg 1$ the

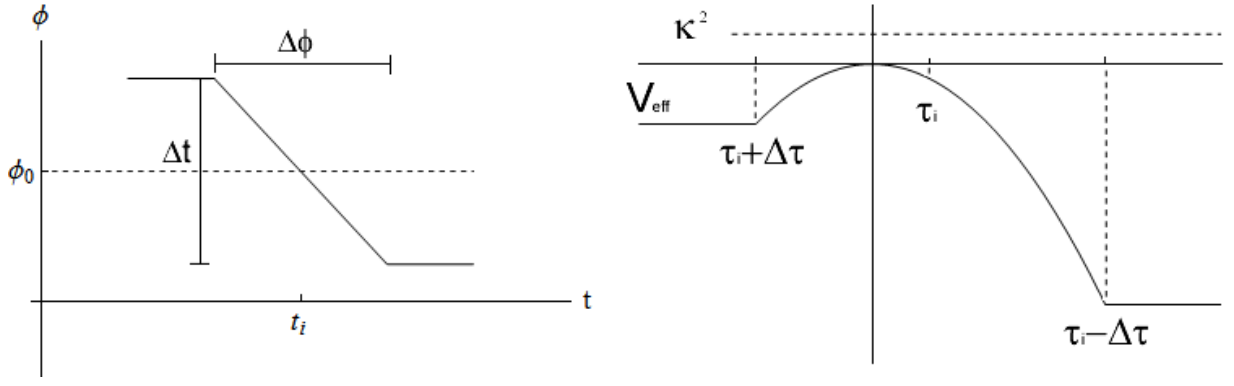


FIG. 6: Left: Linearizing $\phi(t) \approx \phi_0 + \lambda(t - t_i)$ about some point (t_i, ϕ_0) . Right: The associated effective scattering problem.

scattering amplitudes are known [6]:

$$D_k = \frac{e^{i\varphi_k}}{\sqrt{1 + e^{-\pi\kappa^2}}}; \quad (\text{B16})$$

$$R_k = -\frac{ie^{i\varphi_k}}{\sqrt{1 + e^{\pi\kappa^2}}}; \quad (\text{B17})$$

where the angle φ_k is

$$\varphi_k = \arg\Gamma\left(\frac{1 + i\kappa^2}{2}\right) + \frac{\kappa^2}{2} \left(1 + \ln \frac{2}{\kappa^2}\right). \quad (\text{B18})$$

The implications of scattering in this case is considered in more detail in the main text.

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